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Finite Temperature Behavior of the 3D Polyakov Model with Massless Quarks

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Abstract

The (2+1)D Georgi-Glashow (or Polyakov) model with the additional fundamental massless quarks is explored at finite temperature. In the case of vanishing Yukawa coupling, it is demonstrated that the interaction of a monopole and an antimonopole in the molecule via quark zero modes leads to the decrease of the Berezinsky-Kosterlitz-Thouless critical temperature when the number of quark flavors is equal to one. If the number of flavors becomes larger, monopoles are shown to exist only in the molecular phase at any temperatures exceeding a certain exponentially small one. This means that for such a number of flavors and at such temperatures, no fundamental matter can be confined by means of the monopole mechanism.

3D Georgi-Glashow model (else called the Polyakov model) is known to be one of the eldest and the most famous examples of theories allowing for an analytical description of confinement [1]. However, the phase structure of this model at finite temperature has been addressed only recently. Namely, first in Ref. [2] it has been shown that at the temperature $T_c = g^2/2\pi$ the weakly coupled monopole plasma in this model undergoes the Berezinsky-Kosterlitz-Thouless (BKT) [3] phase transition into the molecular phase. Then, in Ref. [4], it has been shown that approximately at the twice smaller temperature, there occurs another phase transition associated to the deconfinement of W-bosons.

In this paper, we shall be interested in the finite-temperature properties of the monopole ensemble, rather than the ensemble of W-bosons. Because of that, let us first discuss in some more details the nature of the above-mentioned BKT phase transition. At high enough temperature, one can apply the idea of dimensional reduction. The dimensionally-reduced theory is then the 2D XY-model, but with the temperature-depending strength of the monopole-antimonopole ($M\bar{M}$)

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interaction. Due to this fact, the phase structure of the model becomes reversed with respect to that of the usual 2D XY-model. Namely, at the temperatures below T_c, monopoles exist in the plasma phase, that leads to the confinement of fundamental matter [1, 5]. At $T > T_c$, the vacuum state is the molecular gas of bound $M\bar{M}-$pairs, and consequently fundamental quarks are deconfined [2]. The analogy with the 2D XY-model established in Ref. [6] is that spin waves of the 2D XY-model correspond to the free photons of the Polyakov model, while vortices correspond to magnetic monopoles.

Let us briefly discuss the BKT phase transition, occuring at $T = T_c$, in the language of the 2D XY-model. At $T < T_c$, the spectrum of the model is dominated by massless spin waves, and the periodicity of the angular variable is unimportant in this phase. The spin waves are unable to disorder the spin-spin correlation functions, and those decrease at large distances by a certain power law. On the contrary, at $T > T_c$, the periodicity of the angular variable becomes important. This leads to the appearance of topological singularities (vortices) of the angular variable, which, contrary to spin waves, have nonvanishing winding numbers. Such vortices condense and disorder the spin-spin correlation functions, so that those start decreasing exponentially with the distance. Thus, the nature of the BKT phase transition is the condensation of vortices at $T > T_c$. In another words, at $T > T_c$, there exist free vortices, which mix in the ground state (vortex condensate) of indefinite global vorticity. Contrary to that, at $T < T_c$, free vortices cannot exist, but they rather mutually couple into bound states of vortex-antivortex pairs. Such vortex-antivortex molecules are small-sized short-living (virtual) objects. Their dipole-type fields are short-ranged and therefore cannot disorder significantly the spin-spin correlation functions. However, when the temperature starts rising, the sizes of these molecules increase, until at $T = T_c$ they diverge, that corresponds to the dissociation of the molecules into pairs. Therefore, coming back to the Polyakov model, one of the methods (which will be employed below) to determine there the critical temperature of the BKT phase transition is to evaluate the mean squared separation in the $M\bar{M}-$molecule and find the temperature at which it starts diverging.

In this paper, we shall consider the extension of the Polyakov model by the fundamental dynamical quarks, which are supposed to be massless. As it will be demonstrated, quark zero modes in the monopole field lead to the additional attraction between a monopole and an antimonopole in the molecule at high temperatures. In particular, when the number of these modes (equal to the number of massless flavors) is sufficiently large, the molecule shrinks so that its size becomes of the order of the inverse W-boson mass. Another factor which governs the size of the molecule is the characteristic range of localization of zero modes. Namely, it can be shown that the stronger zero modes are localized in the vicinity of the monopole center, the smaller molecular size is. In this paper, we shall consider the case when the Yukawa coupling vanishes, and originally massless quarks do not acquire any mass. This means that zero modes are maximally delocalized. Such a weakness of the quark-mediated interaction of monopoles opens a possibility for molecules to undergo eventually the phase transition into the plasma phase. However, this will be shown to occur only provided that the number of flavors is equal to one, whereas at any larger number of flavors, the respective critical temperature becomes exponentially small. This means that the interaction mediated by such a number of zero modes is already strong enough to maintain the molecular phase at any temperature larger than that one.

Let us start our analysis with considering the Euclidean action of the Polyakov model extended by the fundamental, originally massless quarks. [Note that such a model can be viewed as the (2+1)-QCD with the additional adjoint Higgs field.] We shall first consider the general case with the nonvanishing Yukawa coupling, by means of which quarks acquire a certain mass. The action under discussion then reads

$$S = \int d^3x \left[\frac{1}{4}\left(F_{\mu\nu}^a\right)^2 + \frac{1}{2}\left(D_\mu \Phi^a\right)^2 + \frac{\lambda}{4}\left(\left(\Phi^a\right)^2 - \eta^2\right)^2 - i\bar{\psi}\left(\vec{\gamma}\vec{D} + h\frac{\tau^a}{2}\Phi^a\right)\psi\right]. \qquad (1)$$

Here,

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\varepsilon^{abc} A^b_\mu A^c_\nu,\ D_\mu \Phi^a = \partial_\mu \Phi^a + g\varepsilon^{abc} A^b_\mu \Phi^c,\ D_\mu \psi = \left(\partial_\mu - ig\frac{\tau^a}{2} A^a_\mu\right)\psi,$$

and $\bar{\psi} = \psi^\dagger \beta$ with the Euclidean Dirac matrices defined as $\vec{\gamma} = -i\beta\vec{\alpha}$, where

$$\beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\quad \vec{\alpha} = \begin{pmatrix} 0 & \vec{\tau} \\ \vec{\tau} & 0 \end{pmatrix}.$$

Next, in 3D, the electric coupling g, the Yukawa coupling h, and the vacuum expectation value of the Higgs field η have the dimensionality $[\text{mass}]^{1/2}$. The Higgs coupling λ has the dimensionality [mass]. The masses of the W- and Higgs bosons are large compared to g^2 in the standard perturbative (else called weak-coupling) regime $g \ll \eta$ and read: $m_W = g\eta$, $m_H = \eta\sqrt{2\lambda}$. The inequality $g \ll \eta$ is necessary to ensure the spontaneous symmetry breaking from $SU(2)$ to $U(1)$. Note also that for the sake of simplicity, we omit the summation over the flavor indices, but consider the general case with an arbitrary number of flavors.

One can further see that the Dirac equation in the field of the third isotopic component of the 't Hooft-Polyakov monopole [7] decomposes into two equations for the components of the $SU(2)$-doublet ψ. The masses of these components stemming from such equations are equal to each other and read $m_q = h\eta/2$. Next, the Dirac equation in the full monopole potential has been shown [8] to possess the zero mode, whose asymptotic behavior at $r \equiv |\vec{x}\,| \gg m_q^{-1}$ has the following form:

$$\chi^+_{\nu\, n} = \mathcal{N}\frac{\mathrm{e}^{-m_q r}}{r}\left(s^+_\nu s^-_n - s^-_\nu s^+_n\right),\quad \chi^-_{\nu\, n} = 0. \tag{2}$$

Here, $\chi^\pm_n$ are the upper and the lower components of the mode, *i.e.* $\psi = \binom{\chi^+_n}{\chi^-_n}$, next $n = 1, 2$ is the isotopic index, $\nu = 1, 2$ is the Dirac index, $s^+ = \binom{1}{0}$, $s^- = \binom{0}{1}$, and $\mathcal{N}$ is the normalization constant.

It is a well known fact that in 3D, the 't Hooft-Polyakov monopole is actually an instanton [1, 5]. Therefore, we can use the results of Ref. [9] on the quark contribution to the effective action of the instanton-antiinstanton molecule in QCD. Let us thus recapitulate the analysis of Ref. [9] adapting it to our model. To this end, we fix the gauge $\Phi^a = \eta\delta^{a3}$ and define the analogue of the free propagator S_0 by the relation $S_0^{-1} = -i\left(\vec{\gamma}\vec{\partial} + m_q\tau^3\right)$. Next, we define the propagator S_M in the field of a monopole located at the origin, $\vec{A}^{a\,M}$ $[A_i^{a\,M} \to \varepsilon^{aij}x^j/\left(gr^2\right)$ at $r \gg m_W^{-1}]$, by the formula $S_M^{-1} = S_0^{-1} - g\vec{\gamma}\frac{\tau^a}{2}\vec{A}^{a\,M}$. Obviously, the propagator $S_{\bar{M}}$ in the field of an antimonopole located at a certain point $\vec{R}$, $\vec{A}^{a\,\bar{M}}(\vec{x}) = -\vec{A}^{a\,M}\left(\vec{x} - \vec{R}\right)$, is defined by the equation for S_M^{-1} with the replacement $\vec{A}^{a\,M} \to \vec{A}^{a\,\bar{M}}$. Finally, one can consider the molecule made out of these monopole and antimonopole and define the total propagator S in the field of such a molecule, $\vec{A}^a = \vec{A}^{a\,M} + \vec{A}^{a\,\bar{M}}$, by means of the equation for S_M^{-1} with $\vec{A}^{a\,M}$ replaced by $\vec{A}^a$.

One can further introduce the notation $|\psi_n\rangle$, $n = 0, 1, 2, \ldots$, for the eigenfunctions of the operator $-i\vec{\gamma}\vec{D}$ defined at the field of the molecule, namely $-i\vec{\gamma}\vec{D}\,|\psi_n\rangle = \lambda_n\,|\psi_n\rangle$, where $\lambda_0 = 0$. This yields the following formal spectral representation for the total propagator S:

$$S(\vec{x}, \vec{y}) = \sum_{n=0}^{\infty}\frac{|\psi_n(\vec{x})\rangle\,\langle\psi_n(\vec{y})|}{\lambda_n - im_q\tau^3}.$$

Next, it is convenient to employ the mean-field approximation, according to which zero modes dominate in the quark propagator, *i.e.*,

$$S\left(\vec{x},\vec{y}\right)\simeq\frac{\left|\psi_0(\vec{x})\right\rangle\left\langle\psi_0(\vec{y})\right|}{-im_q\tau^3}+S_0\left(\vec{x},\vec{y}\right). \tag{3}$$

Indeed, this approximation is valid, since in the weak-coupling regime the monopole sizes, equal to m_W^{-1}, are much smaller than the average distance in the $M\bar{M}-$plasma. This average distance has an order of magnitude $\zeta^{-1/3}$ (see *e.g.* Ref. [10] for a discussion). Here, $\zeta\propto \mathrm{e}^{-4\pi m_W\epsilon/g^2}$ stands for the so-called monopole fugacity, which has the dimensionality $[\mathrm{mass}]^3$, and $\epsilon\sim 1$ is a certain dimensionless function of (m_H/m_W). Obviously, ζ is exponentially small in the weak-coupling regime under study. The approximation (3) remains valid for the molecular phase near the phase transition (*i.e.* when the temperature approaches the critical one from above), we shall be interested in. That is merely because in this regime, molecules become very much inflated being about to dissociate.

Within the notations adapted, one now has $S=\left(S_M^{-1}+S_{\bar{M}}^{-1}-S_0^{-1}\right)^{-1}=S_{\bar{M}}\mathcal{S}^{-1}S_M$, where

$$\mathcal{S}=S_0-\left(S_M-S_0\right)S_0^{-1}\left(S_{\bar{M}}-S_0\right)=S_0-\frac{\left|\psi_0^M\right\rangle\left\langle\psi_0^M\right|}{-im_q\tau^3}S_0^{-1}\frac{\left|\psi_0^{\bar{M}}\right\rangle\left\langle\psi_0^{\bar{M}}\right|}{-im_q\tau^3},$$

and $\left|\psi_0^M\right\rangle$, $\left|\psi_0^{\bar{M}}\right\rangle$ are the zero modes of the operator $-i\vec{\gamma}\vec{D}$ defined at the field of a monopole and an antimonopole, respectively. Denoting further $a=\left\langle\psi_0^{\bar{M}}\left|g\vec{\gamma}\frac{\tau^a}{2}\vec{A}^{a\,M}\right|\psi_0^M\right\rangle$, it is straightforward to see by the definition of the zero mode that $a=\left\langle\psi_0^{\bar{M}}\left|\left(-i\vec{\gamma}\vec{\partial}\right)\right|\psi_0^M\right\rangle=\left\langle\psi_0^{\bar{M}}\left|S_0^{-1}\right|\psi_0^M\right\rangle$. This yields $\mathcal{S}=S_0+(a^*/m_q^2)\left|\psi_0^M\right\rangle\left\langle\psi_0^{\bar{M}}\right|$, where the star stands for the complex conjugation, and therefore $\det\mathcal{S}=\left[1+\left(|a|/m_q\right)^2\right]\cdot\det S_0$. Finally, defining the desired effective action as $\Gamma=\ln[\det S^{-1}/\det S_0^{-1}]$, we obtain for it in the general case with N_f flavors the following expression: $\Gamma=\text{const}+N_f\ln\left(m_q^2+|a|^2\right)$. The constant in this formula, standing for the sum of effective actions defined at the monopole and at the antimonopole, cancels out in the normalized expression for the mean squared separation in the $M\bar{M}-$molecule.

Let us further set h equal to zero, and so m_q is equal to zero as well. Notice first of all that although in this case the direct Yukawa interaction of the Higgs bosons with quarks is absent, they keep interacting with each other via the gauge field. Owing to this fact, the problem of finding a quark zero mode in the monopole field is still valid [1]. The dependence of the absolute value of the matrix element a on the distance R between a monopole and an antimonopole can now be straightforwardly found. Indeed, we have $|a|\propto\int d^3r/\left(r^2\left|\vec{r}-\vec{R}\right|\right)=-4\pi\ln(\mu R)$, where μ stands for the IR cutoff.

Now we switch on the temperature $T\equiv\beta^{-1}$, so that all the bosonic (fermionic) fields should be supplied with the periodic (antiperiodic) boundary conditions in the temporal direction, with the period equal to β. The magnetic-field lines of a single monopole thus cannot cross the boundary of the one-period region and should go parallel to this boundary at the distances larger than β. Therefore, monopoles separated by such distances interact via the 2D Coulomb law, rather than

[1] Note that according to Eq. (2) this mode will be non-normalizable in the sense of a discrete spectrum. However, in the gapless case $m_q=0$ under discussion, the zero mode, which lies exactly on the border of the two contiguous Dirac seas, should clearly be treated not as an isolated state of a discrete spectrum, but rather as a state of a continuum spectrum. (A similar treatment of the zero mode of a massless left-handed neutrino on electroweak Z-strings has been discussed in Ref. [11].) This means that it should be understood as follows: $\left|\psi_0(\vec{x})\right\rangle\sim\lim\limits_{p\to 0}\left(\mathrm{e}^{ipr}/r\right)$, where $p=|\vec{p}|$. Once being considered in this way, zero modes are normalizable by the standard condition of normalization of the radial parts of spherical waves, R_{pl}, which reads [12] $\int\limits_0^\infty drr^2R_{p'l}R_{pl}=2\pi\delta(p'-p)$.

the 3D one. Recalling that the average distance between monopoles is of the order of $\zeta^{-1/3}$, we conclude that at $T \geq \zeta^{1/3}$, the monopole ensemble becomes two-dimensional (see *e.g.* Ref. [2] for a detailed discussion of the dimensional reduction in the Polyakov model). However, at the temperatures below the exponentially small one, $\zeta^{1/3}$, monopoles keep interacting by the usual 3D Coulomb law, and the monopole confinement mechanism for the fundamental matter works at such temperatures under any circumstances.

We are now in the position to explore a possible modification of the standard BKT critical temperature [2] $T_c = g^2/2\pi$ due to the zero-mode mediated interaction. As it was discussed above, this can be done upon the evaluation of the mean squared separation in the $M\bar{M}-$molecule and further finding the temperature below which it starts diverging. In this way we should take into account that in the dimensionally-reduced theory, the usual Coulomb interaction of monopoles [2] $R^{-1} = \sum\limits_{n=-\infty}^{+\infty} \left(\mathcal{R}^2 + (\beta n)^2\right)^{-1/2}$ goes over into $-2T\ln(\mu\mathcal{R})$, where $\mathcal{R}$ denotes the absolute value of the 2D vector $\vec{\mathcal{R}}$. This statement can be checked *e.g.* by virtue of the Euler - Mac Laurin formula. As far as the novel logarithmic interaction, $\ln(\mu R) = \sum\limits_{n=-\infty}^{+\infty} \ln\left[\mu\left(\mathcal{R}^2 + (\beta n)^2\right)^{1/2}\right]$, is concerned, it transforms into

$$\pi T\mathcal{R} + \ln\left[1 - \exp(-2\pi T\mathcal{R})\right] - \ln 2. \tag{4}$$

Let us prove this statement. To this end, we employ the following formula [13]:

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + x^2} = \frac{1}{2x}\left[\pi\coth(\pi x) - \frac{1}{x}\right].$$

This yields

$$x\sum_{n=-\infty}^{+\infty} \frac{1}{x^2 + (2\pi n/a)^2} = \frac{1}{x} + \frac{xa^2}{2\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2 + (xa/2\pi)^2} = \frac{a}{2}\coth\left(\frac{ax}{2}\right).$$

On the other hand, the L.H.S. of this expression can be written as

$$\frac{1}{2}\frac{d}{dx}\sum_{n=-\infty}^{+\infty}\ln\left(x^2 + \left(\frac{2\pi n}{a}\right)^2\right).$$

Integrating over x with the constant of integration set to zero, we get

$$\sum_{n=-\infty}^{+\infty}\ln\left(x^2 + \left(\frac{2\pi n}{a}\right)^2\right) = a\int dx\coth\left(\frac{ax}{2}\right) = 2\ln\sinh\left(\frac{ax}{2}\right) = ax + 2\ln\left(1 - \mathrm{e}^{-ax}\right) - 2\ln 2.$$

Setting $2\pi/a = \mu\beta$ and $x = \mu\mathcal{R}$ we arrive at Eq. (4).

Thus, the statistical weight of the quark-mediated interaction in the molecule at high temperatures has the form $\exp(-2N_f\ln|a|) \propto \left[\pi T\mathcal{R} + \ln\left[1 - \exp(-2\pi T\mathcal{R})\right] - \ln 2\right]^{-2N_f}$. Accounting for both (former) logarithmic and Coulomb interactions, we eventually arrive at the following expression for the mean squared separation $\left\langle L^2\right\rangle$ in the molecule as a function of T, g, and N_f:

[2] Without the loss of generality, we consider the molecule with the temporal component of $\vec{R}$ equal to zero.

$$\left\langle L^2\right\rangle = \frac{\int\limits_{m_W^{-1}}^{\infty} d\mathcal{R}\mathcal{R}^{3-\frac{8\pi T}{g^2}}\left[\pi T\mathcal{R}+\ln\left[1-\exp(-2\pi T\mathcal{R})\right]-\ln 2\right]^{-2N_f}}{\int\limits_{m_W^{-1}}^{\infty} d\mathcal{R}\mathcal{R}^{1-\frac{8\pi T}{g^2}}\left[\pi T\mathcal{R}+\ln\left[1-\exp(-2\pi T\mathcal{R})\right]-\ln 2\right]^{-2N_f}}.$$

In this equation, we have put the lower limit of integration equal to the inverse mass of the W-boson, which acts as an UV cutoff.

At large $\mathcal{R}$, $\ln 2 \ll \pi T\mathcal{R}$ and $\left|\ln\left[1-\exp(-2\pi T\mathcal{R})\right]\right| \simeq \exp(-2\pi T\mathcal{R}) \ll \pi T\mathcal{R}$. Consequently, we see that $\left\langle L^2\right\rangle$ is finite at $T > T_c = (2-N_f)g^2/4\pi$, that reproduces the standard result [2] at $N_f = 0$. For $N_f = 1$, the plasma phase is still present at $T < g^2/4\pi$, whereas for $N_f \geq 2$ the monopole ensemble may exist only in the molecular phase at any temperature larger than $\zeta^{1/3}$. Clearly, at $N_f \gg \max\left\{1, 4\pi T/g^2\right\}$, $\sqrt{\langle L^2\rangle} \to m_W^{-1}$, which means that such a large number of zero modes shrinks the molecule to the minimal admissible size. Note finally that both the obtained critical temperature $g^2/4\pi$ and the standard one (in the absence of quarks), $g^2/2\pi$, are obviously much larger than $\zeta^{1/3}$, that fully validates the idea of dimensional reduction.

In conclusion of this paper, we have found the critical temperature of the monopole BKT phase transition in the weak-coupling regime of the Polyakov model extended by the massless dynamical quarks, which interact with the Higgs boson only via the gauge field. It has been shown that for $N_f = 1$, this temperature becomes twice smaller than the one in the absence of quarks, whereas for $N_f \geq 2$ it becomes exponentially small, namely of the order of $\zeta^{1/3}$. The latter effect means that this number of quark zero modes, which strengthen the attraction of a monopole and an antimonopole in the molecule, becomes enough for the support of the molecular phase at any temperature exceeding that exponentially small one. Therefore, for $N_f \geq 2$, no fundamental matter (including dynamical quarks themselves) can be confined at such temperatures by means of the monopole mechanism.

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